

MATH 2 - JUNE 2006 - 1

Roll No. 165077

Total No. of Pages : 3

BT 2/M06

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Mathematics – II (2005-06)

Paper : MATH-102 E

Time : Three Hours]

[Maximum Marks : 100

Note :- Attempt FIVE questions in all selecting at least ONE question from each unit. Each question carries equal marks.

UNIT-I

1. (i) Reduce the following matrix into its normal form and hence find its rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

10

- (ii) Find the values of λ for which the equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$$

$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$$

$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

are consistent, and find the ratios of $x : y : z$ when λ has the smallest of these values. What happens when λ has the greater of these values ?

10

2. (i) State and prove Cayley-Hamilton Theorem. 10

(ii) If $S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$, where $a = e^{2\pi i/3}$, prove that $S^{-1} = \frac{1}{3} \bar{S}$.

10

MATH 2 - JUNE 2006 - 2

UNIT-II

3. (i) Solve : $y dx - x dy + 3x^2 y^2 e^{x^3} dx = 0$ 10

- (ii) Find the Orthogonal Trajectories of the family of confocal conics

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, \lambda \text{ being the parameter.} \quad 10$$

4. (i) Solve:

$$x^2 \cdot \frac{d^2 y}{dx^2} + x \cdot \frac{dy}{dx} + y = \log x \cdot \sin(\log x). \quad 10$$

- (ii) A pendulum of length ℓ has one end of string fastened to a peg on a smooth plane inclined at an angle α to the horizon. With the string and the weight on the plane, its time of oscillation is t seconds. If a pendulum of length ℓ' Oscillates in one second when suspended vertically, prove that $\alpha = \sin^{-1}(\ell / \ell' t^2)$. 10

UNIT-III

5. (i) Find the differential equation of all planes which are at a constant distance a from the origin. 10

- (ii) Use Charpit's method to solve the partial differential equation : $pxy + pq + qy = yz$. 10

6. (i) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6 \cdot e^{-3x} \quad 10$$

- (ii) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = x, \quad 0 \leq x \leq 50 \\ = 100 - x, \quad 50 \leq x \leq 100$$

Find the temperature $u(x, t)$ at any time.

10

MATH 2 - JUNE 2006 - 3

UNIT-IV

7. (i) Prove that

$$\int_0^{\infty} \frac{e^{-t} \cdot \sin^2 t}{t} dt = \frac{1}{4} \log 5 \quad 10$$

- (ii) Apply convolution theorem to evaluate :

$$L^{-1} \left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right] \quad 10$$

8. (i) Solve :

$$\frac{d^2 x}{dt^2} - t \cdot \frac{dx}{dt} + x = 1, \quad x(0) = 1, \quad x'(0) = 2 \quad 10$$

- (ii) A Cantilever beam is clamped at the end $x = 0$ and is free at the end $x = \ell$. It carries a uniform load w per unit length from $x = 0$ to $x = \ell/2$. Calculate the deflection y at any point. 10